EFFECT OF DEPOSITS WITH RECTANGULAR PROFILE ON THE THERMAL EFFICIENCY OF LONGITUDINAL RIBS

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On the basis of the solution of the two-dimensional problem of thermal conductivity the article analyzes the effect of deposits with different thicknesses and thermal conductivities on the thermal effectiveness of longitudinal ribs.

The range of application of ribbed surfaces increases substantially as new production methods come into use. An example confirming this are the results of the complex work of the E. O. Paton Institute of Electric Welding and of the Institute of Technical Thermophysics, Academy of Sciences of the UkrSSR, in devising novel heating surfaces. The technology of producing pipes with welded-on ribbing, devised at the Institute of Electric Welding, and the testing and detailed investigation of variegated profiles of such pipes, carried out at the Institute of Thermophysics, made it possible for branch enterprises and plants for the first time to replace smooth pipes by ribbed ones in many installations with elevated temperatures of the heat carriers. It became possible to use ribbed pipes in heat recovery installations for waste gases, in regenerators of gas turbines, in fuel oil heaters, in economizers of steam generators, etc., i.e., under conditions associated with the contamination of heating surfaces.

In evaluating the effectiveness of ribs under such operating conditions it is very important to take the effect of this and similar factors in the form of corrosion formations into account. In addition to contamination, ribbed surfaces may have protective coatings preventing their corrosion. Such coatings are also made from polymers which create additional thermal resistance to the thermal flux, or from metals which improve the emission of heat from the ribbed surfaces.

Together with the accumulation of experimental material on this problem, it is also of great interest to work out analytical methods of calculating contaminated or protected ribs that would make it possible to evaluate the effect of the principal factors on the efficiency of the ribs.

The problem of heat transfer through an uncoated rib is usually viewed as a one-dimensional problem because the thermal conductivity of the rib is determined on the assumption that the temperature gradient across the rib is negligibly small, and that there is no emission of heat at all from the end face of the rib. The evaluation of the tolerated errors is determined from the solution of the two-dimensional problem. The solution of the two-dimensional problem for longitudinal ribs with rectangular profile [1] made it possible to obtain the following expression for the efficiency:

$$\eta = \sum_{i=1}^{\infty} \frac{2\mathrm{Bi}}{\mu_i(\mu_i^2 + \mathrm{Bi} + \mathrm{Bi}^2)} \left[\frac{\mu_i \,\mathrm{th}\left(\mu_i \frac{2h}{\delta_0}\right) + \mathrm{Bi}}{\mu_i + \mathrm{Bi}\,\mathrm{th}\left(\mu_i \frac{2h}{\delta_0}\right)} \right], \tag{1}$$

where μ_i are the roots of the equation $\mathrm{ctg}\,\mu_i = \mu_i/\mathrm{Bi}$; $\mathrm{Bi} = \alpha\delta_0/2\lambda_1$ is the Biot number.

When coated ribs are examined, especially if the thermal conductivities of the rib and the coating differ greatly from each other, the assumption that the temperature field is one-dimensional is not justified, and it becomes necessary to examine the two-dimensional problem of thermal conductivity. At present the effect of coatings and contamination, which as a rule insulate the ribs, is taken into account on the basis of observations in operation [2].

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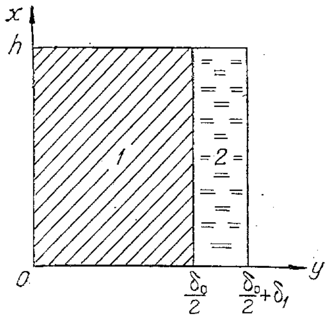

Fig. 1. Diagram for calculating coated ribs: 1) rib; 2) coating of the rib.

In taking into account the coatings, different authors usually proceed from some simplifying assumptions, or they use numerical computer calculation [3-5].

Let us examine the two-dimensional problem of the thermal conductivity of a longitudinal rib with rectangular profile with a uniform coating. Since in practice fairly thin ribs are used, the coating on the end face may be neglected. In the steady-state case, when ribbed surfaces operate in steady regime, the equation of thermal conductivity has the form

$$\frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} = 0, \quad i = 1, 2, \tag{2}$$

with the boundary conditions

$$T_i(x=0) = T_0, \quad \left.\frac{\partial T_i}{\partial x}\right|_{x=h} = 0, \quad \left.\frac{\partial T_1}{\partial y}\right|_{y=0} = 0,$$

$$T_1\left(y = \frac{\delta_0}{2}\right) = T_2\left(y = \frac{\delta_0}{2}\right), \quad \lambda_1 \left.\frac{\partial T_1}{\partial y}\right|_{y=\frac{\delta_0}{2}} = \lambda_2 \left.\frac{\partial T_2}{\partial y}\right|_{y=\frac{\delta_0}{2}}, \tag{3}$$

$$\left[\lambda_2 \frac{\partial T_2}{\partial y} + \alpha (T_2 - T_c)\right]_{y=\delta_1+\frac{\delta_0}{2}} = 0.$$

The subscript i = 1, 2 denotes the rib and the coating, respectively (Fig. 1). For solving the problem we use the method of finite integral transformations [6]. The transformation kernel with respect to x is found from the following Sturm–Liouville problem:

$$\frac{\partial^2 K}{\partial x^2} + \mu^2 K = 0, \quad K(x=0) = 0, \quad \left.\frac{\partial K}{\partial x}\right|_{x=h} = 0. \tag{4}$$

The general solution of problem (4) is written in the form

$$K_n(x) = \frac{\sin(\mu_n x)}{N_n}, \tag{5}$$

where the eigenvalues of μ_n are found from the characteristic equation

$$\cos(\mu_n h) = 0, \quad \mu_n = \frac{\left(n + \frac{1}{2}\right)\pi}{h}, \tag{6}$$

and the normalized coefficient is equal to

$$N_n^2 = \int_0^h \sin^2(\mu_n x)\, dx = \frac{h}{2}\left(1 - \frac{\sin(2\mu_n h)}{2\mu_n h}\right). \tag{7}$$

After transformation and introduction of the new variable $\overline{T}_i^* = \overline{T}_i - \overline{T}_c$, Eq. (2) assumes the form

$$\frac{\partial^2 \overline{T}_i^*}{\partial y^2} - \mu_n^2 \overline{T}_i^* = Q_n, \tag{8}$$

where Q_n is determined by the formula [7]

$$Q_n = \frac{(T_c - T_0)\mu_n}{N_n}. \tag{9}$$

The boundary conditions (3) are transformed in the following way:

$$\left.\frac{\partial \overline{T}_1^*}{\partial y}\right|_{y=0} = 0, \quad \overline{T}_1^*\left(y = \frac{\delta_0}{2}\right) = \overline{T}_2^*\left(y = \frac{\delta_0}{2}\right), \tag{10}$$

$$\lambda_1 \left.\frac{\partial \overline{T}_1^*}{\partial y}\right|_{y=\frac{\delta_0}{2}} = \lambda_2 \left.\frac{\partial \overline{T}_2^*}{\partial y}\right|_{y=\frac{\delta_0}{2}}, \quad \left(\lambda_2 \frac{\partial \overline{T}_2^*}{\partial y} + \alpha \overline{T}_2^*\right)_{y=\frac{\delta_0}{2}+\delta_1} = 0.$$

The general solution of Eq. (8) has the form

$$\overline{T}_i^*(y) = C_{1i}\exp(\mu_n y) + C_{2i}\exp(-\mu_n y) - \frac{Q_n}{\mu_n^2}. \tag{11}$$

The coefficients C_{1i}, C_{2i} are found from the boundary conditions (10) when the following system of equations is solved:

$$C_{11} - C_{21} = 0, \quad C_{11} = C_{21} = C_0,$$

$$C_{11}\exp\left(\mu_n\frac{\delta_0}{2}\right) + C_{21}\exp\left(-\mu_n\frac{\delta_0}{2}\right) = C_{12}\exp\left(\mu_n\frac{\delta_0}{2}\right) + C_{22}\exp\left(-\mu_n\frac{\delta_0}{2}\right),$$

$$\lambda_1\mu_n\left[C_{11}\exp\left(\mu_n\frac{\delta_0}{2}\right) - C_{21}\exp\left(-\mu_n\frac{\delta_0}{2}\right)\right] = \lambda_2\mu_n\left[C_{12}\exp\left(\mu_n\frac{\delta_0}{2}\right) - C_{22}\exp\left(-\mu_n\frac{\delta_0}{2}\right)\right], \tag{12}$$

$$\lambda_2\mu_n\left[C_{12}\exp\left(\mu_n\left(\frac{\delta_0}{2}+\delta_1\right)\right) - C_{22}\exp\left(-\mu_n\left(\frac{\delta_0}{2}+\delta_1\right)\right)\right] +$$

$$+\alpha\left[C_{12}\exp\left(\mu_n\left(\frac{\delta_0}{2}+\delta_1\right)\right) + C_{22}\exp\left(-\mu_n\left(\frac{\delta_0}{2}+\delta_1\right)\right)\right] = 0.$$

From system (12) we find

$$C_{12n} = \frac{P_n\Phi_n}{L_n}\exp\left(-\mu_n\frac{\delta_0}{2}\right), \qquad C_{22n} = \frac{P_n}{L_n}\exp\left(-\mu_n\frac{\delta_0}{2}\right), \tag{13}$$

$$C_{0n} = -C_{12n}\left[\operatorname{ch}\left(\mu_n\frac{\delta_0}{2}\right) + \frac{\rho_{1n}}{\rho_{2n}}\operatorname{sh}\left(\mu_n\frac{\delta_0}{2}\right)\right]^{-1},$$

where the following notation is introduced

$$\rho_{in} = \frac{\lambda_i\mu_n}{\alpha}, \quad P_n = \frac{Q_n}{\mu_n^2(1+\rho_{2n})}, \quad h_n = -\frac{Q_n}{\mu_n^2},$$

$$\Phi_n = \frac{\left[\frac{\rho_{1n}}{\rho_{2n}}\operatorname{th}\left(\mu_n\frac{\delta_0}{2}\right) + 1\right]}{\left[\frac{\rho_{1n}}{\rho_{2n}}\operatorname{th}\left(\mu_n\frac{\delta_0}{2}\right) - 1\right]}, \tag{14}$$

$$L_n = \frac{1-\rho_{2n}}{1+\rho_{2n}}\exp(-\mu_n\delta_1) - \Phi_n\exp(\mu_n\delta_1).$$

By reconversion we obtain the general solutions for temperature fields:

$$T_1(x, y) = T_c + \sum_{n=0}^{\infty}\frac{\sin(\mu_n x)}{N_n}\left[C_{0n}\operatorname{ch}(\mu_n y) + h_n\right], \tag{15}$$

$$T_2(x, y) = T_c + \sum_{n=0}^{\infty}\frac{\sin(\mu_n x)}{N_n}\left[C_{12n}\exp(\mu_n y) + C_{22n}\exp(-\mu_n y) + h_n\right]. \tag{16}$$

The efficiency of the rib is determined in accordance with the expression [8]

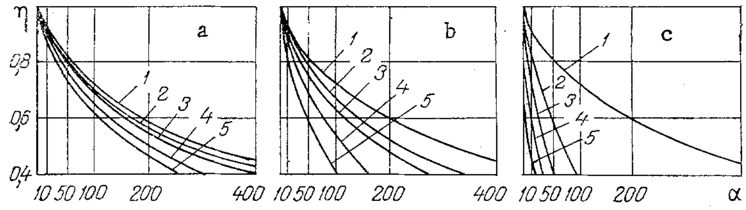

Fig. 2. Dependence of the efficiency η of a longitudinal rib with coating of thickness δ_1 (mm) on the heat transfer coefficient α ($W/m^2 \cdot °C$) with different thermal conductivities of the coating material λ_2 (W/m·°C): λ_1 = 40 W/m·°C, h = 20 mm, δ_0 = 2 mm; a) λ_2 = 0.5; b) 0.1; c) 0.01 W/m·°C; 1) δ_1 = 0; 2) 0.1; 3) 0.2; 4) 0.5; 5) 1.0 mm.

$$\eta = \frac{\int\limits^{l} (T(l) - T_c)\,\alpha(l)\,dl}{(T_0 - T_c)\int\limits^{l} \alpha(l)\,dl}, \tag{17}$$

where integration is carried out over the outer contour of the rib l. In our case

$$\eta = \frac{1}{(T_0 - T_c)\,h}\int\limits_0^h T_2^*\left(x,\ y = \frac{\delta_0}{2} + \delta_1\right)dx. \tag{18}$$

After calculation we obtain the following expression for the efficiency of a coated rib:

$$\eta = \frac{1}{(T_0 - T_c)\,h}\sum_{n=0}^{\infty}\left\{\frac{1}{\mu_n N_n}\left[C_{12n}\exp\left(\mu_n\left(\frac{\delta_0}{2} + \delta_1\right)\right) + C_{22n}\exp\left(-\mu_n\left(\frac{\delta_0}{2} + \delta_1\right)\right) + h_n\right]\right\}. \tag{19}$$

Expressions (15), (16), and (19) enable us to determine the temperature fields and the efficiency of differently coated longitudinal ribs. Figure 2 shows the efficiency of ribs coated with materials with different thermal conductivities and with different thicknesses of the coatings. For comparison we also show the graph of the efficiency of uncoated ribs.

Analysis shows that even a fairly thin contaminating layer with low thermal conductivity of the coating material may substantially impair the efficiency of a longitudinal rib. The thermal conductivity of the coating material and its thickness have approximately the same effect on the change in efficiency.

NOTATION

h, height of the rib; δ_0, thickness of the uncoated rib; δ_1, thickness of the coating; α, heat transfer coefficient of the surface; T_c, ambient temperature; T_0, temperature of the base of the rib; λ_1, thermal conductivity of the material of the rib; λ_2, thermal conductivity of the coating material.

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POSSIBLE EXISTENCE OF PERIODIC TEMPERATURE FIELD DURING ELECTRICAL HEATING

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The temperature distribution in a metal rod heated by electric current is analyzed qualitatively.

Two other studies [1, 2] have dealt with the formation of dissipative three-dimensional structures in nonequilibrium systems. As examples of such structures were considered thermodynamically open mechanical, electrical, hydrodynamical, and chemical systems describable by nonlinear equations. A spatially organized steady structure under heavy thermodynamic fluxes can also form during heat transfer processes.

Just as was done in another study [3], we will here consider the steady temperature field in a metal rod through which there flows an electric current. The differential equation of the temperature field for the case of electrical resistivity $\rho = a + b\vartheta$, with a constant thermal conductivity λ and a heat transfer $q_s(\vartheta)$ which is some function of the temperature head, will be written as

$$\frac{d^2\vartheta}{dz^2} - \frac{\pi}{f\lambda} q_s(\vartheta) + \frac{bI^2}{f^2\lambda}\vartheta + \frac{aI^2}{f^2\lambda} = 0. \tag{1}$$

Equation (1) can be reduced to the autonomous system

$$\frac{d\vartheta}{dz} = y, \tag{2}$$

$$\frac{dy}{dz} = \frac{bI^2}{f^2\lambda}\left[\frac{f\pi}{bI^2} q_s(\vartheta) - \frac{a}{b} - \vartheta\right] = \frac{bI^2}{f^2\lambda} Q(\vartheta).$$

The simple state of equilibrium of system (2) is determined by the condition

$$y = 0, \quad Q(\vartheta) = \frac{f\pi}{bI^2} q_s(\vartheta) - \frac{a}{b} - \vartheta = 0, \quad \frac{dQ(\vartheta)}{d\vartheta} = 0. \tag{3}$$

It is evident from the expression for the roots of the characteristic equation of system (2)

$$k_{1,2} = \pm \frac{bI^2}{f^2\lambda}\left(\frac{dQ}{d\vartheta}\right)^{0,5}\Bigg|_{\vartheta=\vartheta_0} \tag{4}$$

that equilibrium states of both the saddle kind and the center kind can exist [4]. We have a saddle when

$$\frac{dQ}{d\vartheta}\Bigg|_{\vartheta=\vartheta_0} > 0, \tag{5}$$

and a center, i.e., a periodic $\vartheta(z)$ relation, when

$$\frac{dQ}{d\vartheta}\Bigg|_{\vartheta=\vartheta_0} > 0 \tag{6}$$

A complex state of equilibrium, with both roots of the characteristic equation equal to zero at points of standstill, cannot exist in system (2). Indeed, simultaneous vanishing of $Q(\vartheta)$ and $dQ/d\vartheta$ is equivalent to the differential equation

$$\frac{dq_s(\vartheta)}{d\vartheta} = \frac{b}{a + b\vartheta} q_s(\vartheta) \tag{7}$$

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